HERGET'S METHOD INITIATION AND TEST CASE
SPECIFICATION WORKSHEET

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This worksheet defines a test case for the HMC worksheet, which implements Herget's method of preliminary orbit determination.

Define constants and set the Mathcad worksheet ORIGIN to 1 so that subscripts start at unity rather than at zero.

$DegPerRad := \frac{180}{\pi}$ $\mathbf{ORIGIN} \equiv 1$

$SecPerDeg := 3600.0$

$SecPerRev := SecPerDeg \cdot 360.0$

1. Specify the number of observations for this test case, then estimate ρ_1 and ρ_n.

Start with 1.0 A.U., but if the worksheet HMC does not converge with these starting estimates, come back to this worksheet, HM1, and to this step to define better starting estimates.

$n := 5$

$\rho_1 := 1.0$ $\rho_n := 1.0$

2. Specify time (**JDT**), right ascension (**RA**), and declination (**DEC**) for observations 1 through n. Note that time is terrestrial (TT) and that **RA** and **DEC** are referred to the mean equator and equinox of J2000.0 (2000 January 1.5, or JDT 2451545.0).

$JDT_1 := 2450834.74164$

$JDT_2 := 2450840.71590$

$JDT_3 := 2450841.77069$

$JDT_4 := 2450857.56861$

$JDT_5 := 2450885.59222$

$$RA_1 := \frac{15 \cdot \left(03 + \frac{44}{60} + \frac{50.43}{3600}\right)}{DegPerRad} \qquad RA_1 = 0.98105175$$

$$RA_2 := \frac{15 \cdot \left(03 + \frac{45}{60} + \frac{46.10}{3600}\right)}{DegPerRad} \qquad RA_2 = 0.98510019$$

$$RA_3 := \frac{15 \cdot \left(03 + \frac{46}{60} + \frac{02.02}{3600}\right)}{DegPerRad} \qquad RA_3 = 0.98625793$$

$$RA_4 := \frac{15 \cdot \left(03 + \frac{53}{60} + \frac{21.41}{3600}\right)}{DegPerRad} \qquad RA_4 = 1.01821127$$

$$RA_5 := \frac{15 \cdot \left(04 + \frac{18}{60} + \frac{34.54}{3600}\right)}{DegPerRad} \qquad RA_5 = 1.12824919$$

$$DEC_1 := \frac{42 + \frac{12}{60} + \frac{41.6}{3600}}{DegPerRad} \qquad DEC_1 = 0.73673063$$

$$DEC_2 := \frac{41 + \frac{43}{60} + \frac{02.5}{3600}}{DegPerRad} \qquad DEC_2 = 0.72810531$$

$$DEC_3 := \frac{41 + \frac{38}{60} + \frac{02.5}{3600}}{DegPerRad} \qquad DEC_3 = 0.72665087$$

$$DEC_4 := \frac{40 + \frac{32}{60} + \frac{49.2}{3600}}{DegPerRad} \qquad DEC_4 = 0.70767865$$

$$DEC_5 := \frac{39 + \frac{20}{60} + \frac{22.9}{3600}}{DegPerRad} \qquad DEC_5 = 0.68660719$$

3. Specify the observer's ECI geographical coordinates in MPC format and use Newcomb's equation for the right ascension of Greenwich, as a function of time elapsed since January 0.0 UTC, to calculate the observer's ECI positions at the observation times. Note that θ_{Go} is for 1998 January 0.0 UTC (JD 2450813.5).

$RCOS := 0.77837$ $\quad RSIN := 0.62625$ $\quad \lambda_{SEN} := \dfrac{255.1189}{DegPerRad}$

$JDU_o := 2450813.5$ $\quad TTUT := \dfrac{63.184}{86400}$

$$\theta_G(JDU) := \mathrm{mod}\left(\frac{99.45957}{DegPerRad} + \frac{360.98564735}{DegPerRad} \cdot (JDU - JDU_o), 2 \cdot \pi\right)$$

$$SENPOS(k, RC, RS, \lambda) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| \begin{array}{l} \theta \leftarrow \theta_G\left(JDT_i - TTUT\right) \\ R^{\langle i \rangle} \leftarrow \begin{bmatrix} RC \cdot \cos(\theta + \lambda) \\ RC \cdot \sin(\theta + \lambda) \\ RS \end{bmatrix} \end{array} \right. \\ R \end{array} \right.$$

$$R_{SEN} := SENPOS\left(n, RCOS, RSIN, \lambda_{SEN}\right)$$

$$R_{SEN} = \begin{bmatrix} -0.16504186 & -0.11993553 & -0.38479802 & 0.36001134 & -0.1160068 \\ 0.76067144 & 0.76907433 & 0.67660205 & 0.69010991 & 0.76967674 \\ 0.62625 & 0.62625 & 0.62625 & 0.62625 & 0.62625 \end{bmatrix}$$

4. Calculate **L**, **A**, **D** triad for observations 1 through n.

$$LVALUES(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| L^{\langle i \rangle} \leftarrow \begin{bmatrix} \cos\left(DEC_i\right) \cdot \cos\left(RA_i\right) \\ \cos\left(DEC_i\right) \cdot \sin\left(RA_i\right) \\ \sin\left(DEC_i\right) \end{bmatrix} \right. \\ L \end{array} \right.$$

$L := LVALUES(n)$

$$L = \begin{bmatrix} 0.41192221 & 0.41261493 & 0.41242816 & 0.39884919 & 0.33120336 \\ 0.61555733 & 0.62202611 & 0.6233099 & 0.64678145 & 0.69889469 \\ 0.67186998 & 0.66545656 & 0.66437021 & 0.65007159 & 0.63391684 \end{bmatrix}$$

$$AVALUES(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| A^{\langle i \rangle} \leftarrow \begin{bmatrix} -\sin\left(RA_i\right) \\ \cos\left(RA_i\right) \\ 0 \end{bmatrix} \right. \\ A \end{array} \right.$$

$A := AVALUES(n)$

$$A = \begin{bmatrix} -0.83108276 & -0.83332748 & -0.83396689 & -0.8511705 & -0.9036638 \\ 0.55614876 & 0.55277963 & 0.55181448 & 0.52488931 & 0.42824261 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$DVALUES(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| D^{\langle i \rangle} \leftarrow \begin{bmatrix} -\sin\left(DEC_i\right) \cdot \cos\left(RA_i\right) \\ -\sin\left(DEC_i\right) \cdot \sin\left(RA_i\right) \\ \cos\left(DEC_i\right) \end{bmatrix} \right. \\ D \end{array} \right.$$

$D := DVALUES(n)$

$$D = \begin{bmatrix} -0.37365966 & -0.36785083 & -0.3666091 & -0.34121563 & -0.2714702 \\ -0.55837956 & -0.55454324 & -0.55406276 & -0.55332176 & -0.5728477 \\ 0.74066911 & 0.74643658 & 0.74740366 & 0.75987296 & 0.77340122 \end{bmatrix}$$

5. Calculate sun's ECI positions at the observation times. Here we will use **C**, **U2PM** and **HGEO** as defined in previous worksheets, for convenience, even though UPM is not really needed for the very-nearly-circular orbit of the Earth around the sun.

We will need function **C** to calculate the first four c-functions for function **U2PM**.

$$
C(x) := \begin{array}{|l}
N \leftarrow 0 \\
\text{while } |x| \ge 0.1 \\
\quad \begin{array}{|l} x \leftarrow \dfrac{x}{4} \\ N \leftarrow N+1 \end{array} \\
c_3 \leftarrow \dfrac{\left(1-\dfrac{x}{20}\cdot\left(1-\dfrac{x}{42}\cdot\left(1-\dfrac{x}{72}\cdot\left(1-\dfrac{x}{110}\cdot\left(1-\dfrac{x}{156}\cdot\left(1-\dfrac{x}{210}\right)\right)\right)\right)\right)\right)}{6} \\
c_2 \leftarrow \dfrac{\left(1-\dfrac{x}{12}\cdot\left(1-\dfrac{x}{30}\cdot\left(1-\dfrac{x}{56}\cdot\left(1-\dfrac{x}{090}\cdot\left(1-\dfrac{x}{132}\cdot\left(1-\dfrac{x}{182}\right)\right)\right)\right)\right)\right)}{2} \\
c_1 \leftarrow 1 - c_3 \cdot x \\
c_0 \leftarrow 1 - c_2 \cdot x \\
\text{while } N > 0 \\
\quad \begin{array}{|l} N \leftarrow N-1 \\ c_3 \leftarrow \dfrac{\left(c_1 \cdot c_2 + c_3\right)}{4} \\ c_2 \leftarrow \dfrac{c_1 \cdot c_1}{2} \\ c_1 \leftarrow c_1 \cdot c_0 \\ c_0 \leftarrow 2 \cdot c_0 \cdot c_0 - 1 \end{array} \\
\begin{bmatrix} c_0 & c_1 & c_2 & c_3 \end{bmatrix}^{\mathrm{T}}
\end{array}
$$

We will need the uniform, two-body path propagator function, **U2PM**, which will be invoked by function **HGEO**.

$k := 0.01720209895$ $\mu := 1.0$

$K := k \cdot \sqrt{\mu}$

$$
U2PM(K, q, e, i, \Omega, \omega, \Delta t) :=
\begin{array}{l}
\alpha \leftarrow K^2 \cdot \dfrac{(1-e)}{q} \\
p \leftarrow q \cdot (1+e) \\
s \leftarrow \dfrac{\Delta t}{q} \\
\Delta s \leftarrow s \\
\text{while } |\Delta s| \geq 0.00000001 \\
\quad c \leftarrow C(\alpha \cdot s^2) \\
\quad f \leftarrow q \cdot s + K^2 \cdot e \cdot s^3 \cdot c_4 - \Delta t \\
\quad Df \leftarrow q + K^2 \cdot e \cdot s^2 \cdot c_3 \\
\quad DDf \leftarrow K^2 \cdot e \cdot s \cdot c_2 \\
\quad \text{if } Df \geq 0 \\
\quad\quad m \leftarrow 1 \\
\quad \text{else} \\
\quad\quad m \leftarrow -1 \\
\quad \Delta s \leftarrow \dfrac{-5 \cdot f}{\left(Df + m \cdot \sqrt{\left|(4 \cdot Df)^2 - 20 \cdot f \cdot DDf\right|}\right)} \\
\quad s \leftarrow s + \Delta s \\
P_1 \leftarrow \cos(\Omega) \cdot \cos(\omega) - \sin(\Omega) \cdot \cos(i) \cdot \sin(\omega) \\
P_2 \leftarrow \sin(\Omega) \cdot \cos(\omega) + \cos(\Omega) \cdot \cos(i) \cdot \sin(\omega) \\
P_3 \leftarrow \sin(i) \cdot \sin(\omega) \\
Q_1 \leftarrow -(\cos(\Omega) \cdot \sin(\omega) + \sin(\Omega) \cdot \cos(i) \cdot \cos(\omega)) \\
Q_2 \leftarrow -(\sin(\Omega) \cdot \sin(\omega) - \cos(\Omega) \cdot \cos(i) \cdot \cos(\omega)) \\
Q_3 \leftarrow \sin(i) \cdot \cos(\omega) \\
c \leftarrow C(\alpha \cdot s^2) \\
rcosv \leftarrow q - K^2 \cdot s^2 \cdot c_3 \\
rsinv \leftarrow K \cdot \sqrt{p} \cdot s \cdot c_2 \\
rcosv \cdot P + rsinv \cdot Q
\end{array}
$$

We will need function **HGEO** to calculate the ECI equatorial J2000.0 coordinates of the sun. (HGEO actually calculates the heliocentric ecliptic coordinates of the Earth-moon barycenter, and corrects them to the geocenter.)

$$HGEO(JD) := \left| \begin{array}{l} JD_o \leftarrow 2451545.0 \\ T_c \leftarrow \dfrac{JD - JD_o}{36525.0} \\ a \leftarrow 1.00000011 - 0.00000005 \cdot T_c \\ e \leftarrow 0.01671022 - 0.00003804 \cdot T_c \\ q \leftarrow a \cdot (1 - e) \\ \mu \leftarrow 1.00000304 \\ K \leftarrow k \cdot \sqrt{\mu} \\ n \leftarrow K \cdot a^{\frac{-3}{2}} \\ \omega \leftarrow \dfrac{102.94719 + \dfrac{1198.28 \cdot T_c}{SecPerDeg}}{DegPerRad} \\ i \leftarrow \dfrac{0.00005 - \dfrac{46.94 \cdot T_c}{SecPerDeg}}{DegPerRad} \\ \Omega \leftarrow 0.0 \\ L \leftarrow \dfrac{100.46435 + \dfrac{1293740.63 + 99 \cdot SecPerRev}{SecPerDeg} \cdot T_c}{DegPerRad} \\ T \leftarrow JD - \dfrac{\mathrm{mod}(L - \omega, 2 \cdot \pi)}{n} \\ \Delta t \leftarrow JD - T \\ r_{EM} \leftarrow U2PM(K, q, e, i, \Omega, \omega, \Delta t) \\ L_M \leftarrow \dfrac{\mathrm{mod}(218.0 + 481268.0 \cdot T_c, 360.0)}{DegPerRad} \\ \begin{bmatrix} r_{EM_1} - 0.0000312 \cdot \cos(L_M) \\ r_{EM_2} - 0.0000312 \cdot \sin(L_M) \\ r_{EM_3} \end{bmatrix} \end{array} \right.$$

We will need the obliquity of the ecliptic, ε, at J2000.0, in order to transform the ECI ecliptic J2000.0 coordinates of the sun to ECI equatorial J2000.0 coordinates.

$$\varepsilon := \frac{23.4392911}{DegPerRad}$$

$$M := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varepsilon) & -\sin(\varepsilon) \\ 0 & \sin(\varepsilon) & \cos(\varepsilon) \end{bmatrix}$$

$ECEQ(r) := M \cdot r$ (Transforms from ecliptic to equatorial.)

$EQEC(r) := M^{-1} \cdot r$ (Transforms from equatorial to ecliptic.)

$$SUNPOS(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| R^{\langle i \rangle} \leftarrow M \cdot HGEO\left(JDT_i\right) \right. \\ -R \end{array} \right.$$

To obtain the ECI equatorial J2000.0 coordinates of the sun, we compute -1 times the HCI equatorial J2000.0 position vector of Earth (this explains the minus sign of **R** in **SUNPOS**).

$R_{SUN} := SUNPOS(n)$

$$R_{SUN} = \begin{bmatrix} 0.50702917 & 0.59386107 & 0.60853537 & 0.80050416 & 0.98495278 \\ -0.77382133 & -0.72069489 & -0.71046406 & -0.53017468 & -0.12169106 \\ -0.33549748 & -0.31246398 & -0.3080283 & -0.2298621 & -0.05276025 \end{bmatrix}$$

(These values agree well with the Astronomical Almanac for 1998, p. C20.)

6. Calculate the sun's position vectors relative to the observer at the observation times.

$R := R_{SUN} - \frac{R_{SEN}}{23454.79842}$ (The constant is the number of Earth radii in one A.U.)

7. Write Herget's method initiation values to disk for use by the worksheet HMC.

$$\text{WRITEPRN}\left(\text{“RHOVALS.prn”}, \begin{bmatrix} n \\ \rho_1 \\ \rho_n \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$$

Geocentric distance estimates and number of observations.

$$\text{WRITEPRN}\left(\text{“TFILE.prn”}, JDT\right) = \begin{bmatrix} 2450834.74164 \\ 2450840.7159 \\ 2450841.77069 \\ 2450857.56861 \\ 2450885.59222 \end{bmatrix}$$

Observation times.

Values of **L**.

$$\text{WRITEPRN}\left(\text{“LFILE.prn”}, L\right) = \begin{bmatrix} 0.4119222 & 0.4126149 & 0.4124282 & 0.3988492 & 0.3312034 \\ 0.6155573 & 0.6220261 & 0.6233099 & 0.6467814 & 0.6988947 \\ 0.67187 & 0.6654566 & 0.6643702 & 0.6500716 & 0.6339168 \end{bmatrix}$$

Values of **A**.

$$\text{WRITEPRN}\left(\text{“AFILE.prn”}, A\right) = \begin{bmatrix} -0.8310828 & -0.8333275 & -0.8339669 & -0.8511705 & -0.9036638 \\ 0.5561488 & 0.5527796 & 0.5518145 & 0.5248893 & 0.4282426 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Values of **D**.

$$\text{WRITEPRN}\left(\text{“DFILE.prn”}, D\right) = \begin{bmatrix} -0.3736597 & -0.3678508 & -0.3666091 & -0.3412156 & -0.2714702 \\ -0.5583796 & -0.5545432 & -0.5540628 & -0.5533218 & -0.5728477 \\ 0.7406691 & 0.7464366 & 0.7474037 & 0.759873 & 0.7734012 \end{bmatrix}$$

Values of **R**.

$$\text{WRITEPRN}\left(\text{“RFILE.prn”}, R\right) = \begin{bmatrix} 0.5070362 & 0.5938662 & 0.6085518 & 0.8004888 & 0.9849577 \\ -0.7738538 & -0.7207277 & -0.7104929 & -0.5302041 & -0.1217239 \\ -0.3355242 & -0.3124907 & -0.308055 & -0.2298888 & -0.052787 \end{bmatrix}$$

$$\text{WRITEPRN}\left(\text{“RMS.prn”}, \begin{bmatrix} 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Set RMS history to zero.