HERGET'S METHOD INITIATION AND TEST CASE
SPECIFICATION WORKSHEET

Roger L. Mansfield, October 3, 1999
http://astroger.com

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This worksheet defines a test case for the GHC worksheet, which implements Herget's method of preliminary orbit determination for an artificial Earth satellite orbit (i.e., a geocentric orbit).

Define constants and set the Mathcad worksheet ORIGIN to 1 so that subscripts start at unity rather than at zero.

$DegPerRad := \frac{180}{\pi}$

$\mathbf{ORIGIN} \equiv 1$

$SecPerDeg := 3600.0$

$a_e := 6378.135$

$SecPerRev := SecPerDeg \cdot 360.0$

1. Specify the number of observations for this test case, then estimate ρ_1 and ρ_n.

If the worksheet GHC does not converge with these starting estimates, come back to this worksheet, GH1, and to this step to define better starting estimates. (See the note at the end of this worksheet.)

$n := 11$

$\rho_1 := 100$

$\rho_n := 100$

2. Specify time (**JDT**), right ascension (**RA**), and declination (**DEC**) for observations 1 through n. Note that time is terrestrial (TT) and that **RA** and **DEC** are referred to the mean equator and equinox of J2000.0 (2000 January 1.5, or JDT 2451545.0).

$Obs := \text{READPRN}(\text{“OBSERVATIONS.prn”})$

(Retrieve observations matrix from text file OBSERVATIONS.PRN.)

$$Obs = \begin{bmatrix} 1999 & 8 & 18.51899 & 23 & 28 & 23.37 & -5 & 4 & 56.9 & 422 \\ 1999 & 8 & 18.52503 & 23 & 28 & 21.35 & -5 & 4 & 57.3 & 422 \\ 1999 & 8 & 18.54081 & 23 & 28 & 15.69 & -5 & 4 & 56.6 & 422 \\ 1999 & 8 & 18.57286 & 23 & 28 & 3.14 & -5 & 4 & 51.7 & 422 \\ 1999 & 8 & 18.58419 & 23 & 27 & 58.51 & -5 & 4 & 48.9 & 422 \\ 1999 & 8 & 18.59442 & 23 & 27 & 54.32 & -5 & 4 & 45.8 & 422 \\ 1999 & 8 & 19.72619 & 23 & 29 & 3.21 & -5 & 1 & 6.4 & 422 \\ 1999 & 8 & 19.73325 & 23 & 29 & 2.24 & -5 & 1 & 4.8 & 422 \\ 1999 & 8 & 19.74565 & 23 & 29 & 0.47 & -5 & 1 & 2.1 & 422 \\ 1999 & 8 & 19.77046 & 23 & 28 & 57.46 & -5 & 0 & 56.8 & 422 \\ 1999 & 8 & 19.77853 & 23 & 28 & 56.63 & -5 & 0 & 55.2 & 422 \\ 1999 & 8 & 20.66391 & 23 & 29 & 34.89 & -5 & 0 & 47 & 413 \\ 1999 & 8 & 20.66724 & 23 & 29 & 34.49 & -5 & 0 & 47.2 & 413 \\ 1999 & 8 & 20.67062 & 23 & 29 & 34.06 & -5 & 0 & 47 & 413 \\ 1999 & 8 & 20.69003 & 23 & 29 & 31.66 & -5 & 0 & 46.5 & 413 \end{bmatrix}$$

Note that there are 15 astrometric observations consisting of topocentric R.A. and Dec measurements (but we are only using the first 11 in this worksheet).

We define and then invoke the procedural functions **JED19**, **AzRA**, and **ElDec** to extract and convert the **JDT**, **AZRA**, and **ELDEC** observation vectors, respectively, for n observations.

DayCount specifies the count of days from the beginning of the year, up through the last day of the previous month of any non-leap year. **JED19** calculates the number of Julian ephemeris days corresponding to Year, Month, and Day. Note that **JED19** is intended to be used with any Gregorian calendar date since 1900 January 0.0, having JED = 2415019.5.

$$DayCount := \begin{bmatrix} 0 & 31 & 59 & 90 & 120 & 151 & 181 & 212 & 243 & 273 & 304 & 334 \end{bmatrix}^{T}$$

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JED19(Year, Month, Day) := | JED ← 2415019.5
                           | for Y ∈ 1900 .. Year
                           |   | if Y < Year
                           |   |   | if mod(Y, 4) ≠ 0
                           |   |   |   | JED ← JED + 365
                           |   |   | else
                           |   |   |   | if mod(Y, 100) = 0
                           |   |   |   |   | if mod(Y, 400) = 0
                           |   |   |   |   |   | JED ← JED + 366
                           |   |   |   |   | else
                           |   |   |   |   |   | JED ← JED + 365
                           |   |   |   | else
                           |   |   |   |   | JED ← JED + 366
                           |   | else
                           |   |   | JED ← JED + DayCount_Month + Day
                           |   |   | if mod(Y, 4) ≠ 0
                           |   |   |   | JED ← JED + 0
                           |   |   | else
                           |   |   |   | if Month > 2
                           |   |   |   |   | if mod(Y, 100) = 0
                           |   |   |   |   |   | if mod(Y, 400) = 0
                           |   |   |   |   |   |   | JED ← JED + 1
                           |   |   |   |   |   | else
                           |   |   |   |   |   |   | JED ← JED + 0
                           |   |   |   |   | else
                           |   |   |   |   |   | JED ← JED + 1
                           |   |   |   | else
                           |   |   |   |   | JED ← JED + 0
                           | JED
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JDTCalc(k) := | for i ∈ 1 .. k
              |   | JDT_i ← JED19(Obs_{i,1}, Obs_{i,2}, Obs_{i,3})
              | JDT
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$JDT := JDTCalc(n)$

$$AzRA(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| AZRA_i \leftarrow \dfrac{\left(Obs_{i,4} + \dfrac{Obs_{i,5}}{60} + \dfrac{Obs_{i,6}}{3600}\right) \cdot 15}{DegPerRad} \right. \\ AZRA \end{array} \right.$$

$RA := AzRA(n)$ R.A.s are in radians.

$$ElDec(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| \begin{array}{l} ELDEC_i \leftarrow \dfrac{\left|Obs_{i,7}\right| + \dfrac{Obs_{i,8}}{60} + \dfrac{Obs_{i,9}}{3600}}{DegPerRad} \\ \text{if } Obs_{i,7} < 0 \\ \quad \left\| ELDEC_i \leftarrow -ELDEC_i \right. \end{array} \right. \\ ELDEC \end{array} \right.$$

$DEC := ElDec(n)$ Declinations are in radians.

3. Input the observers' geographical coordinates in MPC format and use Newcomb's equation for the right ascension of Greenwich, as a function of time elapsed since January 0.0 UTC, to calculate the observers' ECI positions at the observation times. Note that θ_{Go} is for 1999 January 0.0 UTC (JD 2451178.5).

$Sensor := \text{READPRN}(\text{“SENSORS.prn”})$ $NSen := 2$

$JDU_o := 2451178.5$ $TTUT := \dfrac{64.184}{86400}$

$$\theta_G(JDU) := \text{mod}\left(\frac{99.22086}{DegPerRad} + \frac{360.98564735}{DegPerRad} \cdot (JDU - JDU_o), 2 \cdot \pi\right)$$

Note that TTUT is the time difference, Terrestrial Dynamical Time (TDT) minus Coordinated Universal Time (UTC). The value used here is for 1999, and is composed of 32 leap seconds, (TAI - UTC as of 1999 January 1.0), plus 32.184 seconds (the fixed difference TDT - TAI).

$$SENPOS(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| \begin{array}{l} \theta \leftarrow \theta_G\left(JDT_i - TTUT\right) \\ \text{for } j \in 1..NSen \\ \quad \left\| \begin{array}{l} \text{if } Sensor_{j,1} = Obs_{i,10} \\ \quad \left\| \begin{array}{l} RC \leftarrow Sensor_{j,3} \\ RS \leftarrow Sensor_{j,4} \\ \lambda \leftarrow \dfrac{Sensor_{j,2}}{DegPerRad} \end{array} \right. \end{array} \right. \\ R^{\langle i \rangle} \leftarrow \begin{bmatrix} RC \cdot \cos(\theta + \lambda) \\ RC \cdot \sin(\theta + \lambda) \\ RS \end{bmatrix} \end{array} \right. \\ R \end{array} \right.$$

$$R := -SENPOS(n)$$

$$R^{T} = \begin{bmatrix} -0.47869103 & 0.70847103 & 0.51709 \\ -0.50529837 & 0.68974623 & 0.51709 \\ -0.57126489 & 0.63618608 & 0.51709 \\ -0.68725003 & 0.50868821 & 0.51709 \\ -0.72178086 & 0.45837614 & 0.51709 \\ -0.74980543 & 0.41093567 & 0.51709 \\ -0.80434218 & -0.29001717 & 0.51709 \\ -0.79065068 & -0.32549625 & 0.51709 \\ -0.76283557 & -0.38621004 & 0.51709 \\ -0.69341114 & -0.50025723 & 0.51709 \\ -0.66709087 & -0.53485145 & 0.51709 \end{bmatrix}$$

Display R-transpose to avoid switch in Mathcad Prime 10 worksheet format from Page to Draft.

4. Calculate **L**, **A**, **D** triad for observations 1 through n.

$$LVALUES(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| \; L^{\langle i \rangle} \leftarrow \begin{bmatrix} \cos\left(DEC_i\right) \cdot \cos\left(RA_i\right) \\ \cos\left(DEC_i\right) \cdot \sin\left(RA_i\right) \\ \sin\left(DEC_i\right) \end{bmatrix} \right. \\ L \end{array} \right.$$

$$L := LVALUES(n)$$

$$L^{T} = \begin{bmatrix} 0.98660872 & -0.13694934 & -0.08858959 \\ 0.98658842 & -0.13709425 & -0.08859152 \\ 0.9865322 & -0.13750037 & -0.08858814 \\ 0.98640839 & -0.13840097 & -0.08856448 \\ 0.98636292 & -0.13873326 & -0.08855095 \\ 0.98632192 & -0.13903399 & -0.08853598 \\ 0.98709883 & -0.13410357 & -0.08747643 \\ 0.98709004 & -0.13417329 & -0.0874687 \\ 0.9870739 & -0.1343005 & -0.08745566 \\ 0.9870467 & -0.13451686 & -0.08743007 \\ 0.98703926 & -0.13457653 & -0.08742234 \end{bmatrix}$$

Display L-transpose to avoid switch in Mathcad Prime 10 worksheet format from Page to Draft.

$$AVALUES(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| A^{\langle i \rangle} \leftarrow \begin{bmatrix} -\sin\left(RA_i\right) \\ \cos\left(RA_i\right) \\ 0 \end{bmatrix} \right. \\ A \end{array} \right.$$

$A := AVALUES(n)$

$$A^{T} = \begin{bmatrix} 0.13748993 & 0.99050317 & 0 \\ 0.13763543 & 0.99048296 & 0 \\ 0.13804311 & 0.99042622 & 0 \\ 0.13894697 & 0.99029982 & 0 \\ 0.1392804 & 0.99025298 & 0 \\ 0.13958213 & 0.9902105 & 0 \\ 0.13461962 & 0.99089735 & 0 \\ 0.13468952 & 0.99088785 & 0 \\ 0.13481706 & 0.99087051 & 0 \\ 0.13503395 & 0.99084097 & 0 \\ 0.13509376 & 0.99083282 & 0 \end{bmatrix}$$

Display A-transpose to avoid switch in Mathcad Prime 10 worksheet format from Page to Draft.

$$DVALUES(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| D^{\langle i \rangle} \leftarrow \begin{bmatrix} -\sin\left(DEC_i\right) \cdot \cos\left(RA_i\right) \\ -\sin\left(DEC_i\right) \cdot \sin\left(RA_i\right) \\ \cos\left(DEC_i\right) \end{bmatrix} \right. \\ D \end{array} \right.$$

$D := DVALUES(n)$

$$D^{T} = \begin{bmatrix} 0.08774827 & -0.01218018 & 0.99606821 \\ 0.08774839 & -0.01219333 & 0.99606804 \\ 0.08774001 & -0.01222898 & 0.99606834 \\ 0.08770538 & -0.01230577 & 0.99607045 \\ 0.08768785 & -0.01233341 & 0.99607165 \\ 0.08766926 & -0.01235804 & 0.99607298 \\ 0.08668016 & -0.01177604 & 0.99616659 \\ 0.08667167 & -0.01178112 & 0.99616727 \\ 0.08665724 & -0.01179052 & 0.99616841 \\ 0.08662929 & -0.01180603 & 0.99617066 \\ 0.08662092 & -0.01181021 & 0.99617134 \end{bmatrix}$$

Display D-transpose to avoid switch in Mathcad Prime 10 worksheet format from Page to Draft.

5. Write Herget's method initiation values to disk for use by the worksheet GHC.

$$\text{WRITEPRN}\left(\text{“RHOVALS.prn”}, \begin{bmatrix} n \\ \rho_1 \\ \rho_n \end{bmatrix}\right) = \begin{bmatrix} 11 \\ 100 \\ 100 \end{bmatrix}$$

Topocentric distance estimates and number of observations.

$$\text{WRITEPRN}\left(\text{“TFILE.prn”}, JDT\right) = \begin{bmatrix} 2451409.01899 \\ 2451409.02503 \\ 2451409.04081 \\ 2451409.07286 \\ 2451409.08419 \\ 2451409.09442 \\ 2451410.22619 \\ 2451410.23325 \\ 2451410.24565 \\ 2451410.27046 \\ 2451410.27853 \end{bmatrix}$$

Write out observation times, values of **L**, **A**, **D**, and **R**. For all values of **L**, **A**, and **D**, see transposes above.

$$\text{WRITEPRN}\left(\text{“LFILE.prn”}, L\right) = \begin{bmatrix} 0.98660872 & 0.98658842 & \\ -0.13694934 & -0.13709425 & \\ -0.08858959 & -0.08859152 & \ldots \end{bmatrix}$$

$$\text{WRITEPRN}\left(\text{“AFILE.prn”}, A\right) = \begin{bmatrix} 0.13748993 & 0.13763543 & \\ 0.99050317 & 0.99048296 & \\ 0 & 0 & \ldots \end{bmatrix}$$

$$\text{WRITEPRN}\left(\text{“DFILE.prn”}, D\right) = \begin{bmatrix} 0.08774827 & 0.08774839 & 0.08774001 & \\ -0.01218018 & -0.01219333 & -0.01222898 & \\ 0.99606821 & 0.99606804 & 0.99606834 & \ldots \end{bmatrix}$$

$$\text{WRITEPRN}\left(\text{“RFILE.prn”}, R\right) = \begin{bmatrix} -0.47869103 & -0.50529837 & -0.57126489 & -0.68725003 & \\ 0.70847103 & 0.68974623 & 0.63618608 & 0.50868821 & \\ 0.51709 & 0.51709 & 0.51709 & 0.51709 & \ldots \end{bmatrix}$$

$WRITEPRN(\text{“RMS.prn”}, [0\ \ 0]) = [0\ \ 0]$ Set RMS history to zero.

Notes on the Cassini Earth Flyby Test Case

In order to get the Herget's method worksheet, GHC, to work for this test case, made the following modifications.

1. Modified GH1, Step 1, to use ρ_1 = 100 and ρ_n = 100, rather than ρ_1 = ρ_n = 1.0. Achieved better results with 11 observations rather than with 15.

2. Modified GHC, Step 6, to set $\Delta\rho_1$ = $\Delta\rho_n$ = 0, in order to get Mathcad's Minerr to work for this step.

(See also the GD1/GDC worksheets for the non-linear, batch least squares DC solution for these Cassini observations.)