BATCH LEAST SQUARES DIFFERENTIAL CORRECTION
OF A GEOCENTRIC ORBIT

PART 1 - TEST CASE SPECIFICATION WORKSHEET

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(Updated to PTC's Mathcad Prime 10.0 on 2024 July 26)

This worksheet defines a test case for the GDC worksheet. GDC implements the equations of weighted batch least squares differential correction (DC) of a geocentric orbit.

The test case employs 48 radar (type 3) observations of the hyperbolic Earth 1 flyby trajectory of the Galileo spacecraft, which occurred on 1990 December 8. Of these observations, 31 are from the Millstone Hill radar in Massachusetts and 17 are from the ALTAIR radar on Kwajalein atoll in the Marshall Islands [1].

Here now is an outline of the steps we will follow in this worksheet:

1. Specify the observations and the weight matrix, W.

2. Specify the observers' coordinates and calculate the observer's ECI equatorial position and velocity at each observation time.

3. Calculate the N-by-1 "observed" measurements vector, Y.

4. Specify the initial estimate of state, X_o.

5. Write test case specification values to disk for use by worksheet GDC.

(To create other test cases for GDC, simply duplicate this worksheet, rename as "GD2", "GD3", etc., and modify as desired.)

As a preliminary, we define some constants that we will need and set the Mathcad worksheet ORIGIN to 1 so that subscripts start at unity rather than at zero.

$$DegPerRad := \frac{180}{\pi} \qquad \mathbf{ORIGIN} \equiv 1$$

$a_e := 6378.135$ Earth's mean equatorial radius in km:

1. Specify the observations and the weight matrix, W.

Specify time (**t**), azimuth or right ascension (**AZRA**), and elevation or declination (**ELDEC**) for observations 1 through n. Note that time is reckoned in days since January 0.0 UTC of the year of the observations, and that **RA** and **DEC** are referred to the true equator and mean equinox of date.

$Obs := \mathrm{READPRN}(\text{“GALE1OBS.TXT”})$ (Retrieve observations matrix from text file GALE1OBS.)

$$Obs = \begin{bmatrix}
3 & 369 & 1990 & 342 & 20 & 31 & 12.902 & 100.117 & 9.138 & 3412.5 & -9.70518 & 1 \\
3 & 369 & 1990 & 342 & 20 & 31 & 29.762 & 103.054 & 9.951 & 3252.568 & -9.2552 & 2 \\
3 & 369 & 1990 & 342 & 20 & 31 & 41.192 & 105.219 & 10.479 & 3148.74 & -8.90687 & 3 \\
3 & 369 & 1990 & 342 & 20 & 31 & 57.391 & 108.542 & 11.186 & 3008.883 & -8.34606 & 4 \\
3 & 369 & 1990 & 342 & 20 & 32 & 11.491 & 111.697 & 11.792 & 2895.064 & -7.7869 & 5 \\
3 & 369 & 1990 & 342 & 20 & 32 & 26.761 & 115.416 & 12.429 & 2781.286 & -7.10057 & 6 \\
3 & 369 & 1990 & 342 & 20 & 32 & 43.16 & 119.754 & 13.066 & 2671.565 & -6.26255 & 7 \\
3 & 369 & 1990 & 342 & 20 & 32 & 57.56 & 123.868 & 13.569 & 2587.223 & -5.43778 & 8 \\
3 & 369 & 1990 & 342 & 20 & 33 & 12.91 & 128.551 & 14.018 & 2511.078 & -4.46865 & 9 \\
3 & 369 & 1990 & 342 & 20 & 33 & 28.16 & 133.474 & 14.354 & 2450.825 & -3.4203 & 10 \\
3 & 369 & 1990 & 342 & 20 & 33 & 42.435 & 138.287 & 14.55 & 2409.406 & -2.37356 & 11 \\
3 & 369 & 1990 & 342 & 20 & 33 & 57.133 & 143.389 & 14.613 & 2382.749 & -1.24667 & 12 \\
3 & 369 & 1990 & 342 & 20 & 34 & 12.233 & 148.711 & 14.523 & 2372.858 & -0.0606 & 13 \\
3 & 369 & 1990 & 342 & 20 & 34 & 27.259 & 154.001 & 14.278 & 2380.845 & 1.1219 & 14 \\
3 & 369 & 1990 & 342 & 20 & 34 & 45.06 & 160.143 & 13.79 & 2413.065 & 2.48885 & 15 \\
3 & 369 & 1990 & 342 & 20 & 34 & 56.91 & 164.101 & 13.364 & 2447.755 & 3.35996 & 16 \\
3 & 369 & 1990 & 342 & 20 & 35 & 12.959 & 169.24 & 12.684 & 2510.719 & 4.47111 & 17 \\
3 & 369 & 1990 & 342 & 20 & 35 & 27.709 & 173.686 & 11.967 & 2583.702 & 5.41052 & 18 \\
3 & 369 & 1990 & 342 & 20 & 35 & 42.809 & 177.947 & 11.185 & 2672.11 & 6.28377 & 19 \\
3 & 369 & 1990 & 342 & 20 & 35 & 55.56 & 181.301 & 10.496 & 2756.547 & 6.95007 & 20 \\
3 & 369 & 1990 & 342 & 20 & 36 & 11.861 & 185.269 & 9.601 & 2876.161 & 7.70962 & 21 \\
3 & 369 & 1990 & 342 & 20 & 36 & 26.887 & 188.615 & 8.773 & 2996.711 & 8.32301 & 22 \\
3 & 369 & 1990 & 342 & 20 & 36 & 42.986 & 191.89 & 7.898 & 3135.439 & 8.89598 & 23 \\
3 & 369 & 1990 & 342 & 20 & 36 & 57.187 & 194.539 & 7.134 & 3264.959 & 9.33582 & 24 \\
3 & 369 & 1990 & 342 & 20 & 37 & 12.263 & 197.117 & 6.554 & 3408.853 & 9.74371 & 25 \\
3 & 369 & 1990 & 342 & 20 & 37 & 27.65 & 199.524 & 5.764 & 3561.621 & 10.10375 & 26 \\
3 & 369 & 1990 & 342 & 20 & 37 & 50.158 & 202.683 & 4.673 & 3794.18 & 10.54442 & 27 \\
3 & 369 & 1990 & 342 & 20 & 37 & 59.999 & 203.947 & 4.309 & 3898.753 & 10.70907 & 28 \\
3 & 369 & 1990 & 342 & 20 & 38 & 13.808 & 205.604 & 3.756 & 4048.095 & 10.9149 & 29 \\
3 & 369 & 1990 & 342 & 20 & 38 & 42.231 & 208.651 & 2.556 & 4363.482 & 11.26064 & 30 \\
3 & 369 & 1990 & 342 & 20 & 39 & 14.157 & 211.609 & 1.336 & 4727.887 & 11.55098 & 31 \\
3 & 334 & 1990 & 342 & 20 & 56 & 57.36 & 116.692 & 4.0863 & 14962.12 & 7.06205 & 32 \\
3 & 334 & 1990 & 342 & 20 & 57 & 31.063 & 117.363 & 5.1005 & 15202.15 & 7.18192 & 33 \\
3 & 334 & 1990 & 342 & 20 & 58 & 38.418 & 118.687 & 7.0034 & 15693.46 & 7.40212 & 34 \\
3 & 334 & 1990 & 342 & 20 & 59 & 48.026 & 119.964 & 8.8774 & 16215.9 & 7.60521 & 35 \\
3 & 334 & 1990 & 342 & 21 & 5 & 59.927 & 126.016 & 17.0369 & 19200.38 & 8.36744 & 36 \\
3 & 334 & 1990 & 342 & 21 & 9 & 12.433 & 128.699 & 20.3043 & 20836.15 & 8.61438 & 37 \\
3 & 334 & 1990 & 342 & 21 & 12 & 14.951 & 130.993 & 22.9759 & 22425.13 & 8.78941 & 38 \\
3 & 334 & 1990 & 342 & 21 & 15 & 33.913 & 133.411 & 25.5149 & 24188.78 & 8.93239 & 39 \\
3 & 334 & 1990 & 342 & 21 & 18 & 24.315 & 135.32 & 27.4562 & 25719.2 & 9.02612 & 40 \\
3 & 334 & 1990 & 342 & 21 & 22 & 30.877 & 137.872 & 29.7652 & 27957.86 & 9.12737 & 41 \\
3 & 334 & 1990 & 342 & 21 & 25 & 43.388 & 139.772 & 31.3856 & 29720.82 & 9.18587 & 42 \\
3 & 334 & 1990 & 342 & 21 & 29 & 38.381 & 142.071 & 33.1927 & 31890.69 & 9.23974 & 43 \\
3 & 334 & 1990 & 342 & 21 & 34 & 23.095 & 144.641 & 34.9984 & 34523.8 & 9.28632 & 44 \\
3 & 334 & 1990 & 342 & 21 & 38 & 33.315 & 146.874 & 36.4518 & 36851.26 & 9.31554 & 45 \\
3 & 334 & 1990 & 342 & 21 & 41 & 44.34 & 148.506 & 37.4251 & 38632.44 & 9.33247 & 46 \\
3 & 334 & 1990 & 342 & 21 & 43 & 45.381 & 149.548 & 37.9752 & 39762.59 & 9.34135 & 47 \\
3 & 334 & 1990 & 342 & 21 & 45 & 46.579 & 150.52 & 38.5321 & 40895.21 & 9.34892 & 48
\end{bmatrix}$$

$n := \text{rows}(Obs)$ $\qquad$ $n = 48$

Note that there are 48 radar (type 3) obs consisting of Az, El, Range, and Range Rate measurements. Thus N, the total number of measurements, is N = 4x48 = 192.

We define and then invoke the procedural functions **Time**, **AzRA**, and **ElDec** to extract and convert the **t**, **AZRA**, and **ELDEC** observation vectors, respectively, for n observations.

```
$Time(k) :=$
    for $i \in 1..k$
        $t_i \leftarrow Obs_{i,4} + \frac{Obs_{i,5}}{24} + \frac{Obs_{i,6}}{1440} + \frac{Obs_{i,7}}{86400}$
    $t$
```

$t := Time(n)$ Times are in days since 1996 January 0.0 UTC.

```
$AzRA(k) :=$
    for $i \in 1..k$
        $AZRA_i \leftarrow \frac{Obs_{i,8}}{DegPerRad}$
    $AZRA$
```

$AZRA := AzRA(n)$

Azimuths and R.A.s are in radians.

```
$ElDec(k) :=$
    for $i \in 1..k$
        $ELDEC_i \leftarrow \frac{Obs_{i,9}}{DegPerRad}$
    $ELDEC$
```

$ELDEC := ElDec(n)$

Elevations and Declinations are in radians.

We will also need to specify **OBTYPE** in order to have the observation type for each observation, and will need to extract and convert **RANGE** (range) and **RANGER** (range rate) for each type 3 observation.

```
$ObType(k) :=$
    for $i \in 1..k$
        $Type_i \leftarrow Obs_{i,1}$
    $Type$
```

$OBTYPE := ObType(n)$

```
$Range(k) :=$
    for $i \in 1..k$
        $RANGE_i \leftarrow \frac{Obs_{i,10}}{a_e}$
    $RANGE$
```

```
$RangeRt(k) :=$
    for $i \in 1..k$
        $RANGER_i \leftarrow \frac{Obs_{i,11} \cdot 60}{a_e}$
    $RANGER$
```

$RANGE := Range(n)$

$RANGER := RangeRt(n)$

Ranges are in E.R.

Range Rates are in E.R./min.

Count and output N, the number of measurements in the n observations:

```
$Count(k) :=$
    $j \leftarrow 0$
    for $i \in 1..k$
        if $OBTYPE_i = 5$
            $j \leftarrow j+2$
        if $OBTYPE_i = 3$
            $j \leftarrow j+4$
    $j$
```

$N := Count(n)$

$N = 192$

$\mathrm{WRITEPRN}(\text{"NMEAS.prn"}, N) = [192]$

```
$WEIGHT(k) :=$
    for $i \in 1..k$
        for $j \in 1..k$
            if $i = j$
                $W_{i,j} \leftarrow 1$
            else
                $W_{i,j} \leftarrow 0$
    $W$
```

$W := WEIGHT(N)$

2. Input the observers' geographical coordinates. Use Newcomb's equation for the right ascension of Greenwich, as a function of time elapsed since January 0.0 UTC of the year of interest, to calculate the observer's ECI position at each observation time. Note that θ_{Go} is for 1990 January 0.0 UTC.

$Sensor := \mathrm{READPRN}(\text{"SENSORS.TXT"})$

$NSen := \mathrm{rows}(Sensor)$

$NSen = 2$

$$\theta_G(days) := \mathrm{mod}\left(\frac{99.39796}{DegPerRad} + \frac{360.98564735}{DegPerRad} \cdot days, 2 \cdot \pi\right)$$

We need in **SENPOS** the Earth constant e_e, defined in terms of the Earth constant f.

$f := \frac{1}{298.26}$ Earth's polar vs. equatorial flattening factor.

$e_e := \sqrt{2 \cdot f - f^2}$ Eccentricity of Earth's meridional reference ellipse.

```
$SENPOS(k) :=$
    for $i \in 1..k$
        for $j \in 1..NSen$
            if $Sensor_{j,1} = Obs_{i,2}$
                $\phi \leftarrow \frac{Sensor_{j,2}}{DegPerRad}$
                $\lambda \leftarrow \frac{Sensor_{j,3}}{DegPerRad}$
                $H \leftarrow Sensor_{j,4}$
        $\theta \leftarrow \theta_G\left(t_i\right) + \lambda$
        $G_1 \leftarrow \frac{1}{\sqrt{1 - {e_e}^2 \cdot \sin(\phi)^2}} + \frac{H}{a_e}$
        $G_2 \leftarrow \frac{1 - {e_e}^2}{\sqrt{1 - {e_e}^2 \cdot \sin(\phi)^2}} + \frac{H}{a_e}$
        $R^{\langle i \rangle} \leftarrow \begin{bmatrix} G_1 \cdot \cos(\phi) \cdot \cos(\theta) \\ G_1 \cdot \cos(\phi) \cdot \sin(\theta) \\ G_2 \cdot \sin(\phi) \end{bmatrix}$
    $-R$
```

$R := SENPOS(n)$

$$R^{\mathrm{T}} = \begin{bmatrix} -0.50869258 & 0.53334322 & -0.67361589 \\ -0.50934792 & 0.5327174 & -0.67361589 \\ -0.50979175 & 0.53229268 & -0.67361589 \\ -0.51042017 & 0.53169012 & -0.67361589 \\ -0.51096658 & 0.53116503 & -0.67361589 \\ -0.51155771 & 0.53059574 & -0.67361589 \\ -0.51219185 & 0.52998362 & -0.67361589 \\ -0.51274809 & 0.52944549 & -0.67361589 \\ -0.5133404 & 0.52887122 & -0.67361589 \\ -0.51392821 & 0.52830003 & -0.67361589 \\ -0.51447786 & 0.52776477 & -0.67361589 \\ -0.51504322 & 0.52721306 & -0.67361589 \\ & & \vdots \end{bmatrix}$$

(Show R-transpose.) to avoided forcing worksheet from Page to Draft mode.)

Calculate ECI velocities (we only need for type 3 observations containing range rate measurements, but we will do for all observations).

$$\theta_{dot} := \frac{360.98561229}{DegPerRad \cdot 1440}$$

Earth's rotation rate in rad/min, relative to the fixed equinox.

$$SENVEL(k, R) := \left\| \begin{array}{l} \text{for } i \in 1 \,..\, k \\ \quad \left\| \; V^{\langle i \rangle} \leftarrow \begin{bmatrix} -\theta_{dot} \cdot R_{2,i} \\ \theta_{dot} \cdot R_{1,i} \\ 0 \end{bmatrix} \right. \\ V \end{array} \right.$$

$$V := SENVEL(n, R)$$

There are 48 columns in V.

$$V^{\mathrm{T}} = \begin{bmatrix} -0.00233352 & -0.00222567 & 0 \\ -0.00233078 & -0.00222853 & 0 \\ -0.00232892 & -0.00223048 & 0 \\ -0.00232629 & -0.00223323 & 0 \\ -0.00232399 & -0.00223562 & 0 \\ -0.0023215 & -0.0022382 & 0 \\ -0.00231882 & -0.00224098 & 0 \\ -0.00231647 & -0.00224341 & 0 \\ -0.00231395 & -0.002246 & 0 \\ -0.00231145 & -0.00224857 & 0 \\ -0.00230911 & -0.00225098 & 0 \\ -0.0023067 & -0.00225345 & 0 \\ & & \vdots \end{bmatrix}$$

(Show V-transpose.) to avoided forcing worksheet from Page to Draft mode.)

When we have type 3 observations, we need to calculate and write to disk for use by GDC the **SEZ** matrices, i.e., the topocentric-to-ECI transformation matrices for the type 3 obs. Otherwise, we would need to pass the observers' coordinates to GDC for **SEZ** matrix calculations there. (For algorithmic simplicity, we will calculate **SEZ** matrices for all of the observations.)

$$SEZ(\phi, \theta) := \begin{bmatrix} \sin(\phi) \cdot \cos(\theta) & -\sin(\theta) & \cos(\theta) \cdot \cos(\phi) \\ \sin(\phi) \cdot \sin(\theta) & \cos(\theta) & \sin(\theta) \cdot \cos(\phi) \\ -\cos(\phi) & 0 & \sin(\phi) \end{bmatrix}$$

```
SEZCalc(k) := for i ∈ 1..k
                  for j ∈ 1..NSen
                      if Sensor_{j,1} = Obs_{i,2}
                          $\phi \leftarrow \frac{Sensor_{j,2}}{DegPerRad}$
                          $\lambda \leftarrow \frac{Sensor_{j,3}}{DegPerRad}$
                  $\theta \leftarrow \theta_G(t_i) + \lambda$
                  $M \leftarrow SEZ(\phi, \theta)$
                  if i = 1
                      Table ← M
                  else
                      Table ← stack(Table, M)
              Table
```

$$\mathrm{WRITEPRN}\left(\text{“SEZMATS.prn”}, SEZCalc(n)\right) = \begin{bmatrix} 0.46732561 & 0.72363181 & 0.50790135 \\ -0.48997165 & 0.69018621 & -0.53251364 \\ -0.73589031 & 0 & 0.67710077 \\ 0.46792765 & 0.72278271 & 0.50855566 \\ -0.48939673 & 0.69107536 & -0.5318888 \\ -0.73589031 & 0 & 0.67710077 \\ 0.4683354 & 0.72220646 & 0.50899881 \\ -0.48900655 & 0.69167755 & -0.53146474 \\ -0.73589031 & 0 & 0.67710077 \\ 0.46891271 & 0.72138891 & 0.50962625 \\ -0.48845298 & 0.69253017 & -0.53086311 \\ -0.73589031 & 0 & 0.67710077 \\ 0.46941468 & 0.72067648 & 0.5101718 \\ -0.4879706 & 0.69327153 & -0.53033884 \\ -0.73589031 & 0 & 0.67710077 \\ 0.46995775 & 0.71990407 & 0.51076202 \\ -0.4874476 & 0.69407358 & -0.52977043 \\ -0.73589031 & 0 & 0.67710077 \\ 0.47054032 & 0.71907356 & 0.51139517 \\ -0.48688526 & 0.69493397 & -0.52915927 \\ -0.73589031 & 0 & 0.67710077 \\ 0.47105132 & 0.71834344 & 0.51195054 \\ -0.48639089 & 0.69568866 & -0.52862198 \\ -0.73589031 & 0 & 0.67710077 \\ & & \vdots \end{bmatrix}$$

There are 144 rows in SEZMATS.

3. Calculate the N-by-1 observed measurements vector, Y.

```
$YVALUES(k) :=$
    $N \leftarrow 0$
    for $i \in 1..k$
        if $OBTYPE_i = 5$
            $j \leftarrow N+1$
            $Y_j \leftarrow \cos(ELDEC_i) \cdot AZRA_i$
            $Y_{j+1} \leftarrow ELDEC_i$
            $N \leftarrow N+2$
        if $OBTYPE_i = 3$
            $j \leftarrow N+1$
            $Y_j \leftarrow RANGE_i$
            $Y_{j+1} \leftarrow \cos(ELDEC_i) \cdot AZRA_i$
            $Y_{j+2} \leftarrow ELDEC_i$
            $Y_{j+3} \leftarrow RANGER_i$
            $N \leftarrow N+4$
    $Y$
```

$Y := YVALUES(n)$

(Note that function **YVALUES** converts n observations to N measurements.)

$$Y = \begin{bmatrix} 0.53503101 \\ 1.72519487 \\ 0.15948819 \\ -0.09129797 \\ 0.50995597 \\ 1.77157281 \\ 0.17367771 \\ -0.08706495 \\ 0.49367723 \\ 1.80578953 \\ 0.18289305 \\ -0.08378816 \\ 0.47174966 \\ 1.85842629 \\ 0.19523253 \\ -0.07851254 \\ 0.45390447 \\ 1.90833845 \\ \vdots \end{bmatrix}$$

(Click on column vector and scroll down to see all entries.)

4. Specify the initial estimate of state, $X_{o.}$ We will start with the ECI equatorial values of position and velocity from NASA's nominal elements.

$$r := \begin{bmatrix} 5265.5 \\ -4033.2 \\ 3129.0 \end{bmatrix} \cdot \frac{1}{a_e} \qquad v := \begin{bmatrix} -5.1968 \\ -11.2994 \\ -5.8342 \end{bmatrix} \cdot \frac{60}{a_e}$$

$$X_o := \operatorname{stack}(r, v) \qquad Epoch := 342 + \frac{20}{24} + \frac{34}{1440} + \frac{34.006}{86400}$$

$$X_o = \begin{bmatrix} 0.82555481 \\ -0.63234786 \\ 0.49058228 \\ -0.04888702 \\ -0.10629502 \\ -0.05488313 \end{bmatrix}$$

(Note that the epoch of this state vector is the nominal burnout time specified by NASA, and that the units are E.R. for position and E.R. per minute for velocity.)

5. Write test case specification values to disk for use by worksheet GDC.

$$\text{WRITEPRN}(\text{“NOBS.prn”}, n) = [48]$$

Number of observations.

$$\text{WRITEPRN}(\text{“TVALS.prn”}, t) = \begin{bmatrix} 342.85501044 \\ 342.85520558 \\ 342.85533787 \\ 342.85552536 \\ 342.85568855 \\ 342.85586529 \\ 342.85605509 \\ 342.85622176 \\ 342.85639942 \\ 342.85657593 \\ 342.85674115 \\ 342.85691126 \\ 342.85708603 \\ 342.85725994 \\ 342.85746597 \\ 342.85760313 \\ 342.85778888 \\ 342.85795959 \\ \vdots \end{bmatrix}$$

Observation times.

$$\mathrm{WRITEPRN}\left(\text{“OBTYPES.prn”}, OBTYPE\right)=\begin{bmatrix}3\\3\\3\\3\\3\\3\\3\\3\\3\\3\\3\\3\\\vdots\end{bmatrix}$$

Observation types.

(Number of measurements was written out at end of Step 1.)

Number of measurements, N.

$$\mathrm{WRITEPRN}\left(\text{“WEIGHTS.prn”}, W\right)=\begin{bmatrix}1&0&0&0&0&0&0&0&0&0&0&0&\\0&1&0&0&0&0&0&0&0&0&0&0&\\0&0&1&0&0&0&0&0&0&0&0&0&\\0&0&0&1&0&0&0&0&0&0&0&0&\\0&0&0&0&1&0&0&0&0&0&0&0&\\0&0&0&0&0&1&0&0&0&0&0&0&\\0&0&0&0&0&0&1&0&0&0&0&0&\\0&0&0&0&0&0&0&1&0&0&0&0&\\0&0&0&0&0&0&0&0&1&0&0&0&\\0&0&0&0&0&0&0&0&0&1&0&0&\\0&0&0&0&0&0&0&0&0&0&1&0&\\0&0&0&0&0&0&0&0&0&0&0&1&\\&&&&&&&&&&&&\ddots\end{bmatrix}$$

Values of **R**.

$$\mathrm{WRITEPRN}\left(\text{“RVALS.prn”}, R\right)=\begin{bmatrix}-0.50869258&-0.50934792&-0.50979175&\\0.53334322&0.5327174&0.53229268&\\-0.67361589&-0.67361589&-0.67361589&\cdots\end{bmatrix}$$

Values of **V**.

$$\mathrm{WRITEPRN}\left(\text{“VVALS.prn”}, V\right)=\begin{bmatrix}-0.00233352&-0.00233078&-0.00232892&\\-0.00222567&-0.00222853&-0.00223048&\\0&0&0&\cdots\end{bmatrix}$$

(**SEZ** matrices were written out at the end of Step 2.)

Array of **SEZ** matrices for type 3 obs.

$$\mathrm{WRITEPRN}\left(\text{“YVALS.prn”}, Y\right) = \begin{bmatrix} 0.53503101 \\ 1.72519487 \\ 0.15948819 \\ -0.09129797 \\ 0.50995597 \\ 1.77157281 \\ 0.17367771 \\ -0.08706495 \\ 0.49367723 \\ 1.80578953 \\ 0.18289305 \\ -0.08378816 \\ \vdots \end{bmatrix}$$

Values of Y.

$$\mathrm{WRITEPRN}\left(\text{“STATE.prn”}, X_o\right) = \begin{bmatrix} 0.82555481 \\ -0.63234786 \\ 0.49058228 \\ -0.04888702 \\ -0.10629502 \\ -0.05488313 \end{bmatrix}$$

State vector (corrected by GDC).

$$\mathrm{WRITEPRN}\left(\text{“EPOCH.prn”}, Epoch\right) = \left[342.85733803\right]$$

Epoch of state vector.

$$\mathrm{WRITEPRN}\left(\text{“RMS.prn”}, \left[0 \ \ 0\right]\right) = \left[0 \ \ 0\right]$$

RMS history for state corrections by GDC (one entry for each iteration).

REFERENCE

[1] Mansfield, Roger L., "Algorithms for Reducing Radar Observations of a Hyperbolic Near Earth Flyby" *Journal of the Astronautical Sciences* (April-June 1993), pp. 249-259.

(See Mathcad worksheet GDC for additional references.)