PLANT PHOTOPERIOD VS. DAY NUMBER, AS A FUNCTION OF GEOGRAPHIC LATITUDE

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December 19, 2003

(Updated to PTC's Mathcad Prime 10.0 on 2024 July 25)

This Mathcad worksheet defines a procedure for calculating plant photoperiod, or "the duration of daylight" (time of sunset minus time of sunrise), for each day of the year 2003, as a function of geographic latitude. Photoperiod vs. day number is plotted for geographic latitudes from 5°S to 25°S, in five-degree increments. This analysis was requested by Dr. Lilian B. P. Zaidan, plant physiologist at the Instituto de Botanica, Sao Paulo, Brazil [1].

Actual astronomical data are used. The procedure is based upon equations given in [2].

1. Set up constants particular to the year 2003, as well as other worksheet constants.

$J_Y := 2452639.5$ — Julian date corresponding to 2003 January 0.0 Universal Time (UT).

$J_{1900} := 2415020.0$ — Julian date corresponding to 1900 January 0.5 Ephemeris Time (ET).

$\Delta T_o := \dfrac{65.0}{86400.0}$ — Reduction to ET, in days, at epoch J_Y.

$\mathbf{ORIGIN} \equiv 1$ — Set Mathcad origin to 1 so that vector and matrix subscripts start at 1.

$DegPerRad := \dfrac{180}{\pi}$ — Number of degrees in one radian.

$\alpha_{Go} := \dfrac{99.25166}{DegPerRad}$ — Right ascension of Greenwich at 2003 January 0.0 UT, in radians.

$\omega_E := \dfrac{360.98564735}{DegPerRad}$ — Earth's rotation rate in radians/day.

$R := \dfrac{50}{60} \cdot \dfrac{1}{DegPerRad}$ — Twilight factor (set to its value for sunrise and sunset).

2. Compute Sun's apparent orbital eccentricity (the "ellipticity"), inclination (the "obliquity of the ecliptic"), argument of perigee, and mean anomaly at 2003 January 0.0 ET.

$t_o := J_Y - J_{1900}$ Days elapsed since 1900 January 0.5.

$t := t_o + \Delta T_o$ Convert t_o to Ephemeris Time.

$T := \frac{t}{36525.0}$ Convert t to Julian centuries.

$$e_o := 0.01675104 - 4.180 \cdot 10^{-5} \cdot T - 1.26 \cdot 10^{-7} \cdot T^2$$

$$i_o := \frac{23.452294 - 0.0130125 \cdot T - 1.64 \cdot 10^{-6} \cdot T^2}{DegPerRad}$$

$$\omega_o := \frac{281.22083 + 4.70684 \cdot 10^{-5} \cdot t + 4.53 \cdot 10^{-4} \cdot T^2}{DegPerRad}$$

$$M_o := \frac{358.47583 + 0.985600267 \cdot t - 1.5 \cdot 10^{-4} \cdot T^2}{DegPerRad}$$

3. Define annual constants of simpler procedure for calculating Sun's position [2, p. 57].

$$A_1 := 2 \cdot e_o$$

$$A_2 := \frac{5}{4} \cdot {e_o}^2$$

$$B_1 := \cos\left(i_o\right)$$

$$B_2 := \sin\left(i_o\right)$$

4. Define a function that computes Sun's (α, δ) position on the celestial sphere, as a function of D, the number of days elapsed from January 0.0 of the year of interest to the instant of interest.

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Sun(D) := | M ← M_o + (0.9856·D)/DegPerRad
          | v ← M + A_1·sin(M) + A_2·sin(2·M)
          | u ← mod(v + ω_o, 2·π)
          | α_1 ← atan(|B_1·tan(u)|)
          | if u ≤ π/2
          |   | α ← α_1
          | else
          |   | if u ≤ π
          |   |   | α ← π − α_1
          |   | else
          |   |   | if u ≤ (3/2)·π
          |   |   |   | α ← π + α_1
          |   |   | else
          |   |   |   | α ← 2·π − α_1
          | δ ← asin(B_2·sin(u))
          | [α]
          | [δ]
```

5. Define Photoperiod(D,L), a function that computes photoperiod (duration of daylight) as a function of day number D and latitude L. Use this function to generate photoperiod curves for five southern latitudes of interest.

$$Photoperiod(D, L) := \begin{array}{l} \alpha_G \leftarrow \operatorname{mod}(\alpha_{Go} + \omega_E \cdot D, 2 \cdot \pi) \\ d_R \leftarrow D \\ \text{for } i \in 1..3 \\ \quad RADec \leftarrow Sun(d_R) \\ \quad \alpha \leftarrow RADec_1 \\ \quad \delta \leftarrow RADec_2 \\ \quad \theta \leftarrow \frac{\pi}{2} + \operatorname{asin}\left(\frac{\sin(R) + \sin(L) \cdot \sin(\delta)}{\cos(L) \cdot \cos(\delta)}\right) \\ \quad d_R \leftarrow D + \frac{\operatorname{mod}(\alpha - \alpha_G + 2 \cdot \pi, 2 \cdot \pi) - \theta}{\omega_E} \\ d_S \leftarrow D \\ \text{for } i \in 1..3 \\ \quad RADec \leftarrow Sun(d_S) \\ \quad \alpha \leftarrow RADec_1 \\ \quad \delta \leftarrow RADec_2 \\ \quad \theta \leftarrow \frac{\pi}{2} + \operatorname{asin}\left(\frac{\sin(R) + \sin(L) \cdot \sin(\delta)}{\cos(L) \cdot \cos(\delta)}\right) \\ \quad d_S \leftarrow D + \frac{\operatorname{mod}(\alpha - \alpha_G + 2 \cdot \pi, 2 \cdot \pi) + \theta}{\omega_E} \\ (d_S - d_R) \cdot 1440 \end{array}$$

Define **Curve**, a function that calculates photoperiod curves as functions of geographic latitude.

$$Curve(N, L) := \begin{array}{l} \text{for } i \in 1..N \\ \quad Period_i \leftarrow Photoperiod(i, L) \\ Period \end{array}$$

Calculate photoperiod curves for geographic latitudes 5°S, 10°S, 15°S, 20°S, and 25°S.

$$Curve05S := Curve\left(366, \frac{-5}{DegPerRad}\right)$$

$$Curve10S := Curve\left(366, \frac{-10}{DegPerRad}\right)$$

$$Curve15S := Curve\left(366, \frac{-15}{DegPerRad}\right)$$

$$Curve20S := Curve\left(366, \frac{-20}{DegPerRad}\right)$$

$$Curve25S := Curve\left(366, \frac{-25}{DegPerRad}\right)$$

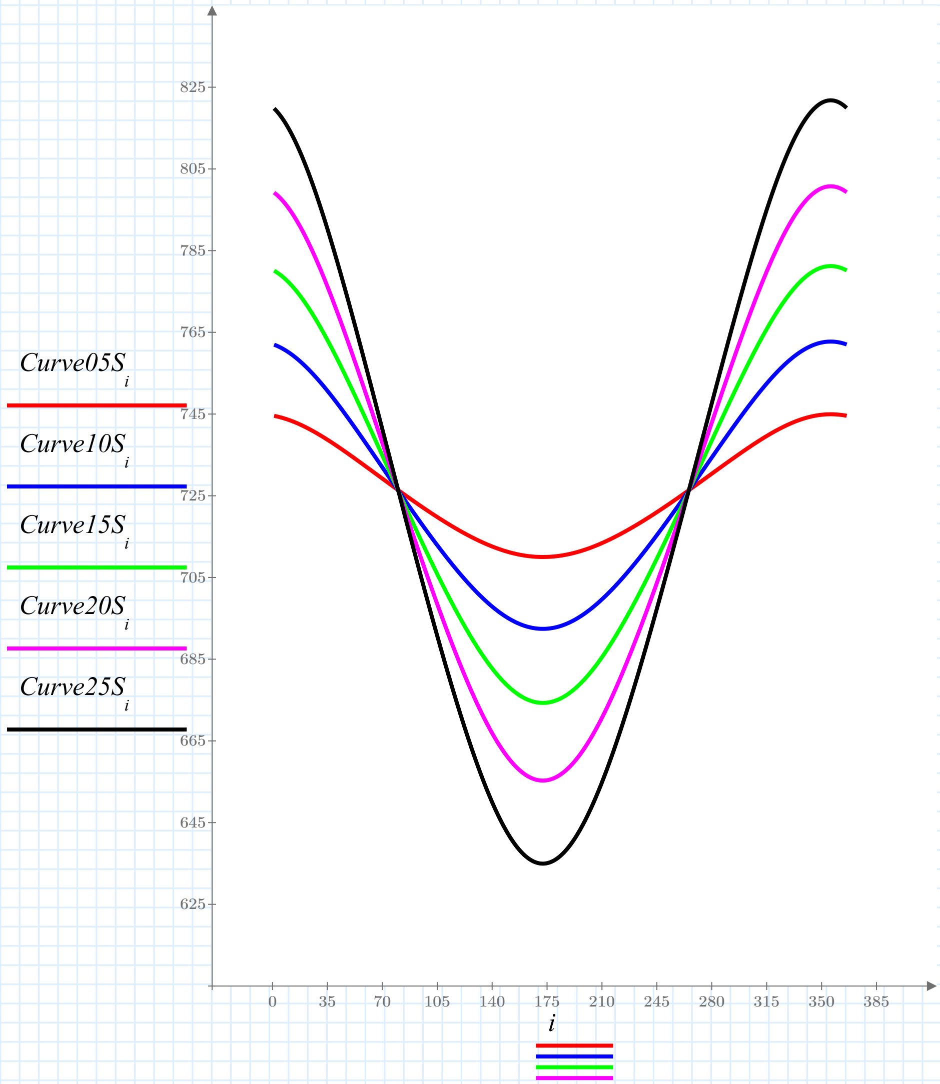
Extract summer and winter values of the photoperiod from the photoperiod curves.

Summer Solstice (Dec 22) Values, in Minutes	Winter Solstice (Jun 21) Values, in Minutes
$\max(Curve05S) = 744.9$	$min(Curve05S) = 710.0$
$\max(Curve10S) = 762.7$	$min(Curve10S) = 692.4$
$\max(Curve15S) = 781.2$	$min(Curve15S) = 674.3$
$\max(Curve20S) = 800.7$	$min(Curve20S) = 655.3$
$\max(Curve25S) = 821.8$	$min(Curve25S) = 635.0$

Mathcad Prime needs
range variable range for plot:

$$i := 1, 2 .. 366$$

Plot the photoperiod curves.



REFERENCES

[1] Zaidan, Lilian B. P., Instituto de Botanica, C.P. 4005, 01061-970 São Paulo, SP. Brazil. Private e-mail communication, December 17, 2003.

[2] Mansfield, Roger L., Sunlight Summary (1979), a 105-page booklet with astronomical equations and a custom-prepared report giving the local times of sunrise, sunset, and the beginning and ending of civil, nautical, and astronomical twilight. Includes appendices on the Gregorian calendar and Milankovitch's astronomical theory of the ice ages. Published by Astronomical Data Service, 3922 Leisure Lane, Colorado Springs, CO 80917-3502 U.S.A.

Addendum: A Simpler Photoperiod Function

The foregoing analysis provides accurate values for the year of interest, and practically speaking, for any other year of interest during the period 1900-2100 (at least).

Yet the analysis does suggest a much simpler way to compute the photoperiod. Here is a much simpler photoperiod function.

$$Photoperiod(D, L) := \left\| \begin{array}{l} RADec \leftarrow Sun(D + 0.5) \\ \delta \leftarrow RADec_2 \\ \theta \leftarrow \dfrac{\pi}{2} + \text{asin}\left(\dfrac{\sin(R) + \sin(L) \cdot \sin(\delta)}{\cos(L) \cdot \cos(\delta)}\right) \\ \dfrac{2 \cdot \theta}{\omega_E} \cdot 1440 \end{array} \right.$$

If this simpler photoperiod function is used instead of the previously defined photoperiod function, then the results only change by about 2-3 minutes.

So here, finally, is a simpler photoperiod calculation that can be worked by hand, and which is accurate to within about 2-3 minutes.

1. Use an astronomical almanac, or some other source, to look up the Sun's declination, δ, at noon on the date for which the photoperiod for a given latitude L is desired.
2. Calculate θ as given in the simpler photoperiod function.
3. Double θ, divide by ω_E, and multiply by 1440 to convert to minutes.

The "Photoperiod Equation" is now seen to be as follows:

$$Photoperiod(\delta, L) := \frac{\pi + 2 \cdot \text{asin}\left(\dfrac{\sin(R) + \sin(L) \cdot \sin(\delta)}{\cos(L) \cdot \cos(\delta)}\right)}{\omega_E} \cdot 1440$$

For example, at the southern hemisphere winter and summer solstices, for which δ = 23.4389 degrees and δ = -23.4389 degrees, respectively, one obtains for latitude 25^{o}S:

$$Photoperiod\left(\frac{23.4389}{DegPerRad}, \frac{-25}{DegPerRad}\right) = 633.1$$

$$Photoperiod\left(\frac{-23.4389}{DegPerRad}, \frac{-25}{DegPerRad}\right) = 819.3$$