BATCH LEAST SQUARES DIFFERENTIAL CORRECTION
OF A GEOCENTRIC ORBIT

PART 1 - TEST CASE SPECIFICATION WORKSHEET

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This worksheet defines a test case for the Gdc worksheet. Gdc implements the equations of weighted batch least squares differential correction (DC) of a geocentric orbit.

The test case employs angles-only (astrometric) observations of the hyperbolic Earth impact trajectory of asteroid 2024 UQ on 2024 October 22. The initial estimate of the path was generated via the Herget's method worksheets Gh1 and Ghc.

Here now is an outline of the steps we will follow in this worksheet:

1. Specify the observations and the weight matrix, W.

2. Specify the observer's coordinates and calculate the observer's ECI equatorial positions at the observation times.

3. Calculate the N-by-1 "observed" measurements vector, Y.

4. Specify the initial estimate of state, X_o.

5. Write test case specification values to disk for use by worksheet Gdc.

(To create other test cases for Gdc, simply duplicate this worksheet, and modify as desired.)

As a preliminary, we define some constants and set the Mathcad worksheet ORIGIN to 1 so that subscripts start at unity rather than at zero.

$DegPerRad := \frac{180}{\pi}$ $\mathbf{ORIGIN} \equiv 1$

$SecPerDeg := 3600.0$ Seconds per degree.

$SecPerRev := SecPerDeg \cdot 360.0$ Seconds per revolution.

$a_e := 6378.137$ Earth's mean equatorial radius in km.

1. Specify the observations and the weight matrix, W.

$n := 8$ $\qquad\qquad$ $N := 2 \cdot n$

Specify time (**t**), right ascension (**RA**), and declination (**DEC**) for observations 1 through n. Note that time is reckoned in days since January 0.0 UTC of the year of the observations, and that **RA** and **DEC** are referred to the mean equator and equinox of J2000.0.

(Retrieve observations matrix from text file "2024 UQ Obs.txt".)

$Obs := \text{READPRN}(\text{"2024 UQ obs.txt"})$

$$Obs = \begin{bmatrix} 2024 & 10 & 22.327039 & 1 & 43 & 1.879 & 13 & 8 & 39.99 & 703 \\ 2024 & 10 & 22.331619 & 1 & 42 & 57.062 & 13 & 8 & 35.24 & 703 \\ 2024 & 10 & 22.333908 & 1 & 42 & 54.459 & 13 & 8 & 31.47 & 703 \\ 2024 & 10 & 22.380923 & 1 & 49 & 3.701 & 13 & 40 & 58.76 & 905 \\ 2024 & 10 & 22.384088 & 1 & 49 & 12.9 & 13 & 42 & 38.63 & 905 \\ 2024 & 10 & 22.385898 & 1 & 49 & 18.458 & 13 & 43 & 38.5 & 905 \\ 2024 & 10 & 22.387174 & 1 & 49 & 22.555 & 13 & 44 & 22.88 & 905 \\ 2024 & 10 & 22.39079 & 1 & 49 & 35.016 & 13 & 46 & 36.95 & 905 \end{bmatrix}$$

Observation data input are, row by row:

Year, Month, Day_of_Month+Fractional_Day,

Right Ascension in hours, minutes, and seconds.

Declination in degrees, minutes, and seconds,

Sensor Number (905 is really MPC code T05).

We define and then invoke functions **Time**, **RtAsc**, and **Decl** to extract and convert **t**, **RA**, and **DEC** observation vectors, respectively, for n observations:

$$Time(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \| \; t_i \leftarrow Obs_{i,3} + 274 \\ t \end{array} \right.$$

(2024 October 0 (September 30) is day 274 since 2000 January 0.0.)

$$RtAsc(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| RA_i \leftarrow \dfrac{\left(Obs_{i,4} + \dfrac{Obs_{i,5}}{60} + \dfrac{Obs_{i,6}}{3600}\right) \cdot 15}{DegPerRad} \right. \\ RA \end{array} \right.$$

$$Decl(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| \begin{array}{l} DEC_i \leftarrow \dfrac{\left|Obs_{i,7}\right| + \dfrac{Obs_{i,8}}{60} + \dfrac{Obs_{i,9}}{3600}}{DegPerRad} \\ \text{if } Obs_{i,7} < 0 \\ \quad \left\| DEC_i \leftarrow -DEC_i \right. \end{array} \right. \\ DEC \end{array} \right.$$

$t := Time(n)$ $RA := RtAsc(n)$ $DEC := Decl(n)$

$$WEIGHT(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| \begin{array}{l} \text{for } j \in 1..k \\ \quad \left\| \begin{array}{l} \text{if } i = j \\ \quad \left\| W_{i,j} \leftarrow 1 \right. \\ \text{else} \\ \quad \left\| W_{i,j} \leftarrow 0 \right. \end{array} \right. \end{array} \right. \\ W \end{array} \right.$$

$W := WEIGHT(N)$

See also the NOTE at end of this worksheet.

$$SNum(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| SNUM_i \leftarrow Obs_{i,10} \right. \\ SNUM \end{array} \right.$$

Sensor numbers
(Observatory codes).

$SNUM := SNum(n)$

2. Input the observers' geographical coordinates in LLH format and use Newcomb's equation for the right ascension of Greenwich, as a function of time elapsed since January 0.0 UTC, to calculate the observers' ECI positions at the observation times. Note that θ_{Go} is for 2000 January 0.0 UTC (Julian Date 2451543.5).

$Sensor := \text{READPRN}(\text{"SENSORS.txt"})$ $\quad NSen := 2$

$$Sensor = \begin{bmatrix} 703 & 32.416944 & -110.733056 & 2.52003 \\ 905 & 20.707627 & -156.257 & 3.04082 \end{bmatrix}$$

$JDU_o := 2451543.5$

$$JDTCalc(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| JDT_i \leftarrow t_i + JDU_o \right. \\ JDT \end{array} \right.$$

$JDT := JDTCalc(n)$

$$\theta_G(JDU) := \text{mod}\left(\frac{98.98215}{DegPerRad} + \frac{360.98564736}{DegPerRad} \cdot (JDU - JDU_o), 2 \cdot \pi\right)$$

Below are two constants related to the shape of Earth's surface, as needed in **SENPOS**.

$f := \frac{1}{298.26}$ Earth's polar vs. equatorial flattening.

$e_e := \sqrt{2 \cdot f - f^2}$ Eccentricity of Earth's reference ellipsoid.

Function **SENPOS** calculates the sensor ECI* positions at the observation times.
(*ECI = Earth-Centered, Inertial)

$$SENPOS(k) := \begin{array}{l} \text{for } i \in 1..k \\ \quad \text{for } j \in 1..NSen \\ \quad\quad \text{if } Sensor_{j,1} = Obs_{i,10} \\ \quad\quad\quad \phi \leftarrow \dfrac{Sensor_{j,2}}{DegPerRad} \\ \quad\quad\quad \lambda \leftarrow \dfrac{Sensor_{j,3}}{DegPerRad} \\ \quad\quad\quad H \leftarrow Sensor_{j,4} \\ \quad \theta \leftarrow \theta_G\left(JDT_i\right) + \lambda \\ \quad G_1 \leftarrow \dfrac{1}{\sqrt{1 - {e_e}^2 \cdot \sin(\phi)^2}} + \dfrac{H}{a_e} \\ \quad G_2 \leftarrow \dfrac{1 - {e_e}^2}{\sqrt{1 - {e_e}^2 \cdot \sin(\phi)^2}} + \dfrac{H}{a_e} \\ \quad R^{\langle i \rangle} \leftarrow \begin{bmatrix} G_1 \cdot \cos(\phi) \cdot \cos(\theta) \\ G_1 \cdot \cos(\phi) \cdot \sin(\theta) \\ G_2 \cdot \sin(\phi) \end{bmatrix} \\ -R \end{array}$$

This version of SENPOS works with latitude, east longitude, and height rather than Minor Planet Center (MPC) coordinates (RC, RS, and longitude).

$$R := SENPOS(n)$$

$$R^T = \begin{bmatrix} -0.66559925 & -0.52109214 & -0.53321254 \\ -0.65028773 & -0.54007893 & -0.53321254 \\ -0.64243158 & -0.54940062 & -0.53321254 \\ -0.91582828 & -0.19440645 & -0.35154792 \\ -0.91176985 & -0.21262887 & -0.35154792 \\ -0.90928586 & -0.22301237 & -0.35154792 \\ -0.90746364 & -0.2303151 & -0.35154792 \\ -0.90198152 & -0.25092757 & -0.35154792 \end{bmatrix}$$

Show values of **R**-transpose, since transpose matrix takes less horizontal worksheet space than **R**.

3. Calculate the N-by-1 observed measurements vector, Y.

$$YVALUES(n) := \text{for } i \in 1..n \quad \left\| \begin{array}{l} j \leftarrow 2 \cdot i - 1 \\ k \leftarrow j + 1 \\ Y_j \leftarrow \cos\left(DEC_i\right) \cdot RA_i \\ Y_k \leftarrow DEC_i \\ Y \end{array} \right.$$

$$Y := YVALUES(n)$$

$$Y = \begin{bmatrix} 0.43778043 \\ 0.22941379 \\ 0.43744165 \\ 0.22939076 \\ 0.43725918 \\ 0.22937248 \\ 0.46236585 \\ 0.23881321 \\ 0.4629612 \\ 0.23929739 \\ 0.46332104 \\ 0.23958765 \\ 0.46358609 \\ 0.23980281 \\ 0.46439244 \\ 0.2404528 \end{bmatrix}$$

(Note that function **YVALUES** converts n observations to N = 2n measurements.)

(Click on column vector and scroll down if necessary to see all 16 entries.)

4. Specify the initial estimate of state, X_o.

We will start with the ECI equatorial values of position and velocity from the Herget's method worksheets Gh1 and Ghc.

$$r := \begin{bmatrix} 208399.34897676 \\ 101849.07822108 \\ 56338.44293589 \end{bmatrix} \qquad v := \begin{bmatrix} -18.5205911 \\ -8.72836619 \\ -4.77538602 \end{bmatrix}$$

Units on left are km for position and km/sec for velocity.

$$r := \frac{r}{a_e} \qquad v := v \cdot \frac{60}{a_e}$$

$$r = \begin{bmatrix} 32.67401578 \\ 15.96846826 \\ 8.83305626 \end{bmatrix} \qquad v = \begin{bmatrix} -0.17422571 \\ -0.08210892 \\ -0.0449227 \end{bmatrix}$$

But we need units of E.R. for position and E.R./min for velocity.

$X_o := \text{stack}(r, v)$

$$X_o = \begin{bmatrix} 32.67401578 \\ 15.96846826 \\ 8.83305626 \\ -0.17422571 \\ -0.08210892 \\ -0.0449227 \end{bmatrix}$$

Note that the epoch of this state vector is the time of the first 2024 UQ observation, and that the units are E.R. for position and E.R. per minute for velocity.

5. Write test case specification values to disk for use by worksheet Gdc.

$\text{WRITEPRN}(\text{“NOBS.prn”}, n) = [8]$ Number of observations.

$$\text{WRITEPRN}(\text{“TVALS.prn”}, t) = \begin{bmatrix} 296.327039 \\ 296.331619 \\ 296.333908 \\ 296.380923 \\ 296.384088 \\ 296.385898 \\ 296.387174 \\ 296.39079 \end{bmatrix}$$

Observation times.

Measurement weights matrix.

$$\text{WRITEPRN}(\text{“WEIGHTS.prn”}, W) = \begin{bmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix}$$

Values of **R** (full **R**-transpose matrix shown above).

$$\text{WRITEPRN}\left(\text{“RVALS.prn”}, R\right) = \begin{bmatrix} -0.66559925 & -0.65028773 & -0.64243158 & \\ -0.52109214 & -0.54007893 & -0.54940062 & \\ -0.53321254 & -0.53321254 & -0.53321254 & \cdots \end{bmatrix}$$

Values of Y.

$$\text{WRITEPRN}\left(\text{“YVALS.prn”}, Y\right) = \begin{bmatrix} 0.43778043 \\ 0.22941379 \\ 0.43744165 \\ 0.22939076 \\ 0.43725918 \\ 0.22937248 \\ 0.46236585 \\ 0.23881321 \\ 0.4629612 \\ 0.23929739 \\ 0.46332104 \\ 0.23958765 \\ 0.46358609 \\ 0.23980281 \\ 0.46439244 \\ 0.2404528 \end{bmatrix}$$

State vector to be corrected by Gdc.

$$\text{WRITEPRN}\left(\text{“STATE.prn”}, X_o\right) = \begin{bmatrix} 32.67401578 \\ 15.96846826 \\ 8.83305626 \\ -0.17422571 \\ -0.08210892 \\ -0.0449227 \end{bmatrix}$$

$$\text{WRITEPRN}\left(\text{“RMS.prn”}, \begin{bmatrix} 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

RMS history for state corrections by Gdc, one entry for each iteration.

NOTE ON 2024 UQ ORBIT DETERMINATION

Astronomical observatories that report observations to the IAU's Minor Planet Center (MPC) take very precise measurements of right ascension and declination.

Simply assuming that the weight matrix W here is the 16x16 identity matrix leads to the correct solution in the batch DC, given these MPC observations.

But the entries in the ATA-inverse matrix of the Batch DC Part 2 worksheet become very large when we assume diagonal weights of unity in the W matrix.

We do not know the calibration results of the many observatories that contribute observations to the MPC.But the right ascension and declination measurements are probably good to better than an arc-second of error.

No matter the statistics, the cartesian state vector of the DC solution predicts that when its atmospheric entry flash was detected on 2024 October 22, asteroid 2024 UQ was at N30 deg. latitude, W136 deg. longitude, and altitude 38.2 km. For more information, see Minor Planet Electronic Circular M.P.E.C. 2024-U49, posted 2024 October 23.