HERGET'S METHOD INITIATION AND TEST CASE
SPECIFICATION WORKSHEET

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This worksheet defines a test case for the Ghc worksheet, which itself implements Herget's method of preliminary orbit determination for asteroid 2024 UQ. Asteroid 2024 UQ impacted Earth on 2024 October 22.

Define constants and set the Mathcad worksheet ORIGIN to 1 so that subscripts start at unity rather than at zero.

$DegPerRad := \frac{180}{\pi}$ $\quad\quad$ $\mathbf{ORIGIN} \equiv 1$

$SecPerDeg := 3600.0$ $\quad\quad$ $a_e := 6378.137$

$SecPerRev := SecPerDeg \cdot 360.0$

1. Specify the number of observations for this test case, then estimate ρ_1 and ρ_n.

If the worksheet Ghc does not converge with these starting estimates, come back to this worksheet, Gh1, and to this step to define better starting estimates. (See the note at the end of this worksheet.)

$n := 9$ $\quad\quad$ $\rho_1 := 10.0$

$\rho_n := 10.0$

Note that ρ_1 and ρ_n have "dot subscripts," and are just "first guesses" of the ranges associated with the first and n-th observations.

2. Input observations and specify time (**JDT**), right ascension (**RA**), and declination (**DEC**) for observations 1 through n. Note that time is terrestrial (TT) and that **RA** and **DEC** are referred to the mean equator and equinox of J2000.0 (2000 January 1.5, or JDT 2451545.0).

$Obs := \text{READPRN}(\text{"2024 UQ Obs.txt"})$

(Retrieve observations matrix from text file 2024 UQ Obs.txt.)

$$Obs = \begin{bmatrix} 2024 & 10 & 22.327039 & 1 & 43 & 1.879 & 13 & 8 & 39.99 & 703 \\ 2024 & 10 & 22.331619 & 1 & 42 & 57.062 & 13 & 8 & 35.24 & 703 \\ 2024 & 10 & 22.333908 & 1 & 42 & 54.459 & 13 & 8 & 31.47 & 703 \\ 2024 & 10 & 22.380923 & 1 & 49 & 3.701 & 13 & 40 & 58.76 & 905 \\ 2024 & 10 & 22.382732 & 1 & 49 & 8.623 & 13 & 41 & 51.83 & 905 \\ 2024 & 10 & 22.384088 & 1 & 49 & 12.9 & 13 & 42 & 38.63 & 905 \\ 2024 & 10 & 22.385898 & 1 & 49 & 18.458 & 13 & 43 & 38.5 & 905 \\ 2024 & 10 & 22.387174 & 1 & 49 & 22.555 & 13 & 44 & 22.88 & 905 \\ 2024 & 10 & 22.39079 & 1 & 49 & 35.016 & 13 & 46 & 36.95 & 905 \end{bmatrix}$$

Observation data input are, row by row:

Year, Month, Day_of_Month+Fractional_Day,

Right Ascension in hours, minutes, and seconds.

Declination in degrees, minutes, and seconds,

Sensor Number (905 is really MPC code T05).

Note that there are nine astrometric observations consisting of topocentric R.A. and Dec measurements.

We define and then invoke the procedural functions **JED19**, **AzRA**, and **ElDec** to extract and convert the **JDT**, **AZRA**, and **ELDEC** observation vectors, respectively, for n observations.

DayCount specifies the count of days from the beginning of the year, up through the last day of the previous month of any non-leap year. **JED19** calculates the number of Julian ephemeris days corresponding to Year, Month, and Day. Note that **JED19** is intended to be used with any Gregorian calendar date since 1900 January 0.0, having JED = 2415019.5.

$$DayCount := \begin{bmatrix} 0 & 31 & 59 & 90 & 120 & 151 & 181 & 212 & 243 & 273 & 304 & 334 \end{bmatrix}^{T}$$

```
JED19(Year, Month, Day) :=
    JED ← 2415019.5
    for Y ∈ 1900 .. Year
        if Y < Year
            if mod(Y, 4) ≠ 0
                JED ← JED + 365
            else
                if mod(Y, 100) = 0
                    if mod(Y, 400) = 0
                        JED ← JED + 366
                    else
                        JED ← JED + 365
                else
                    JED ← JED + 366
        else
            JED ← JED + DayCount_Month + Day
            if mod(Y, 4) ≠ 0
                JED ← JED + 0
            else
                if Month > 2
                    if mod(Y, 100) = 0
                        if mod(Y, 400) = 0
                            JED ← JED + 1
                        else
                            JED ← JED + 0
                    else
                        JED ← JED + 1
                else
                    JED ← JED + 0
    JED
```

```
JDTCalc(k) :=
    for i ∈ 1 .. k
        JDT_i ← JED19(Obs_{i,1}, Obs_{i,2}, Obs_{i,3})
    JDT
```

$JDT := JDTCalc(n)$

$$JDT = \begin{bmatrix} 2460605.827039 \\ 2460605.831619 \\ 2460605.833908 \\ 2460605.880923 \\ 2460605.882732 \\ 2460605.884088 \\ 2460605.885898 \\ 2460605.887174 \\ 2460605.89079 \end{bmatrix}$$

$$AzRA(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| AZRA_i \leftarrow \dfrac{\left(Obs_{i,4} + \dfrac{Obs_{i,5}}{60} + \dfrac{Obs_{i,6}}{3600}\right) \cdot 15}{DegPerRad} \right. \\ AZRA \end{array} \right.$$

$RA := AzRA(n)$ Right ascensions are in radians.

$$ElDec(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| \begin{array}{l} ELDEC_i \leftarrow \dfrac{\left|Obs_{i,7}\right| + \dfrac{Obs_{i,8}}{60} + \dfrac{Obs_{i,9}}{3600}}{DegPerRad} \\ \text{if } Obs_{i,7} < 0 \\ \quad \left\| ELDEC_i \leftarrow -ELDEC_i \right. \end{array} \right. \\ ELDEC \end{array} \right.$$

$DEC := ElDec(n)$ Declinations are in radians.

3. Input the observers' geographical coordinates in latitude/longitude/height (LLH) format and use Newcomb's equation for the right ascension of Greenwich, as a function of time elapsed since January 0.0 UTC, to calculate the observers' ECI positions at the observation times. Note that θ_{Go} is for 2024 January 0.0 UTC (JD 2460309.5).

$Sensor := \text{READPRN}(\text{“SENSORS.txt”})$ $NSen := 2$

$$Sensor = \begin{bmatrix} 703 & 32.416944 & -110.733056 & 2.52003 \\ 905 & 20.707627 & -156.257 & 3.04082 \end{bmatrix}$$

$JDU_o := 2460309.5$

$$\theta_G(JDU) := \text{mod}\left(\frac{99.16697}{DegPerRad} + \frac{360.98564736}{DegPerRad} \cdot (JDU - JDU_o), 2 \cdot \pi\right)$$

Need flattening factor f and e_e for **SENPOS** in LLH format.

$$f := \frac{1}{298.26}$$

Earth's polar vs. equatorial flattening.

$$e_e := \sqrt{2 \cdot f - f^2}$$

Eccentricity of Earth's reference ellipsoid.

$$
SENPOS(k) := \left\| \begin{array}{l}
\text{for } i \in 1..k \\
\quad \left\| \begin{array}{l}
\text{for } j \in 1..NSen \\
\quad \left\| \begin{array}{l}
\text{if } Sensor_{j,1} = Obs_{i,10} \\
\quad \left\| \begin{array}{l}
\phi \leftarrow \dfrac{Sensor_{j,2}}{DegPerRad} \\
\lambda \leftarrow \dfrac{Sensor_{j,3}}{DegPerRad} \\
H \leftarrow Sensor_{j,4}
\end{array} \right.
\end{array} \right. \\
\theta \leftarrow \theta_G\left(JDT_i\right) + \lambda \\
G_1 \leftarrow \dfrac{1}{\sqrt{1 - {e_e}^2 \cdot \sin(\phi)^2}} + \dfrac{H}{a_e} \\
G_2 \leftarrow \dfrac{1 - {e_e}^2}{\sqrt{1 - {e_e}^2 \cdot \sin(\phi)^2}} + \dfrac{H}{a_e} \\
R^{\langle i \rangle} \leftarrow \begin{bmatrix} G_1 \cdot \cos(\phi) \cdot \cos(\theta) \\ G_1 \cdot \cos(\phi) \cdot \sin(\theta) \\ G_2 \cdot \sin(\phi) \end{bmatrix}
\end{array} \right. \\
-R
\end{array} \right.
$$

$$R := SENPOS(n)$$

$$
R^{\mathrm{T}} = \begin{bmatrix}
-0.6639149 & -0.52323646 & -0.53321254 \\
-0.64854221 & -0.54217376 & -0.53321254 \\
-0.64065603 & -0.55147007 & -0.53321254 \\
-0.91519641 & -0.19735964 & -0.35154792 \\
-0.91288763 & -0.20777745 & -0.35154792 \\
-0.91107923 & -0.21556887 & -0.35154792 \\
-0.90856176 & -0.22594431 & -0.35154792 \\
-0.90671599 & -0.23324112 & -0.35154792 \\
-0.9011674 & -0.2538358 & -0.35154792
\end{bmatrix}
$$

Display transposes so that Mathcad Prime does not go from page to draft format.

4. Calculate **L**, **A**, **D** triad for observations 1 through n.

$$LVALUES(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| L^{\langle i \rangle} \leftarrow \begin{bmatrix} \cos\left(DEC_i\right) \cdot \cos\left(RA_i\right) \\ \cos\left(DEC_i\right) \cdot \sin\left(RA_i\right) \\ \sin\left(DEC_i\right) \end{bmatrix} \right. \\ L \end{array} \right.$$

$$L := LVALUES(n)$$

$$L^{\mathrm{T}} = \begin{bmatrix} 0.87704201 & 0.42318258 & 0.22740671 \\ 0.87719492 & 0.4228776 & 0.22738428 \\ 0.87727869 & 0.42271335 & 0.22736648 \\ 0.86366653 & 0.44511164 & 0.23654968 \\ 0.86345303 & 0.44539284 & 0.23679966 \\ 0.86326669 & 0.44563672 & 0.2370201 \\ 0.86302535 & 0.44595401 & 0.23730207 \\ 0.86284707 & 0.44618766 & 0.23751108 \\ 0.86230513 & 0.44689825 & 0.23814242 \end{bmatrix}$$

$$AVALUES(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| A^{\langle i \rangle} \leftarrow \begin{bmatrix} -\sin\left(RA_i\right) \\ \cos\left(RA_i\right) \\ 0 \end{bmatrix} \right. \\ A \end{array} \right.$$

$$A := AVALUES(n)$$

$$A^{\mathrm{T}} = \begin{bmatrix} -0.43456833 & 0.90063887 & 0 \\ -0.43425281 & 0.90079104 & 0 \\ -0.43408228 & 0.90087323 & 0 \\ -0.45811317 & 0.88889388 & 0 \\ -0.45843131 & 0.88872984 & 0 \\ -0.45870771 & 0.88858721 & 0 \\ -0.45906683 & 0.88840173 & 0 \\ -0.45933151 & 0.88826492 & 0 \\ -0.46013625 & 0.88784831 & 0 \end{bmatrix}$$

$$DVALUES(k) := \left\| \begin{array}{l} \text{for } i \in 1..k \\ \quad \left\| D^{\langle i \rangle} \leftarrow \begin{bmatrix} -\sin\left(DEC_i\right) \cdot \cos\left(RA_i\right) \\ -\sin\left(DEC_i\right) \cdot \sin\left(RA_i\right) \\ \cos\left(DEC_i\right) \end{bmatrix} \right. \\ D \end{array} \right.$$

$$D := DVALUES(n)$$

$$D^{\mathrm{T}} = \begin{bmatrix} -0.20481132 & -0.09882375 & 0.97379987 \\ -0.20482572 & -0.09874226 & 0.97380511 \\ -0.20482838 & -0.09869576 & 0.97380926 \\ -0.21026756 & -0.10836653 & 0.97161939 \\ -0.21045093 & -0.10855638 & 0.9715585 \\ -0.21061303 & -0.10872295 & 0.97150475 \\ -0.21081957 & -0.10893751 & 0.97143591 \\ -0.21097276 & -0.10909632 & 0.97138483 \\ -0.21143435 & -0.10957796 & 0.97123024 \end{bmatrix}$$

5. Write Herget's method initiation values to disk for use by the worksheet Ghc.

$$\mathrm{WRITEPRN}\left(\text{“RHOVALS.prn”}, \begin{bmatrix} n \\ \rho_1 \\ \rho_n \end{bmatrix}\right) = \begin{bmatrix} 9 \\ 10 \\ 10 \end{bmatrix}$$

Topocentric distance estimates and number of observations.

Observation times. $\mathrm{WRITEPRN}(\text{“TFILE.prn”}, JDT)$

Values of **L**. $\mathrm{WRITEPRN}(\text{“LFILE.prn”}, L)$

Values of **A**. $\mathrm{WRITEPRN}(\text{“AFILE.prn”}, A)$

Values of **D**. $\mathrm{WRITEPRN}(\text{“DFILE.prn”}, D)$

Values of **R**. $\mathrm{WRITEPRN}(\text{“RFILE.prn”}, R)$

Set RMS history to zero.

$$\mathrm{WRITEPRN}(\text{“RMS.prn”}, [0\ \ 0]) = [0\ \ 0]$$

See also the Gd1/Gdc worksheets for the non-linear, batch least squares DC solution for these 2024 UQ observations.